Abstract

We perform the homogenization of the non-stationary Stokes-Nernst-Planck-Poisson system using two-scale convergence. Due to the specific nonlinear coupling of the underlying equations special attention has to be paid when passing to the two-scale limit. The resulting homogenized system consists of averaged macroscopic equations as well as cell problems that deliver suitable effective parameters. We perform numerical simulations of both the Stokes-Nernst-Planck-Poisson system and the derived fully coupled two-scale problem. The discretization is done in two space dimensions using mixed finite elements. Furthermore, we study the breakthrough curves for the microscopic and the homogenized problem for different choices of the small scale parameter ε.

Microscopic Problem

We regard the distribution of charged colloidal particles within a porous medium. The Nernst-Planck equation describes the transport which includes diffusion, convection and electric drift, and linear reaction between (number-) densities of colloidal particles εj. The electrostatic potential ϕ is given as the solution of Poisson’s equation and the Stokes equations determine the underlying fluid flow of the solution with velocity v, pressure p and viscosity μ. The constants εj, D, μ, ρ and k describe electric permittivity, diffusivity, total charge, temperature and Boltzmann constant, respectively.

Transport takes place within the pore space of a highly heterogeneous porous medium that is de-